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Production asymmetries of D^\pm , Λ_c^+/Λ_c^- and $\Lambda_b^0/\bar{\Lambda}_b^0$ at the LHC from heavy quark recombination mechanism

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The asymmetry in the forward region production cross section of D^\pm is calculated using the heavy quark recombination mechanism for pp collisions at 7 TeV. By suitable choices of four nonperturbative parameters, our calculated results can reproduce those obtained at LHCb. We find $A_p \sim -1\%$ when integrated over $2.0 \text{ GeV} < p_T < 18 \text{ GeV}$ and $2.2 < \eta < 4.75$, which agrees with $A_p = -0.96 \pm 0.26 \pm 0.18\%$ as measured by LHCb. Furthermore, the calculated distributions in η and p_T agree reasonably well with those obtained at LHCb. Using the heavy quark recombination mechanism, we also make predictions on the production asymmetries of Λ_c^+/Λ_c^- and $\Lambda_b^0/\bar{\Lambda}_b^0$ for pp collisions at 7 TeV and 14 TeV in the forward region. We find that the integrated asymmetries for these Λ baryons in the LHCb region are of the order of $\sim 1 - 3\%$ and should be measurable.

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1 Introduction

One of the simplest signals for CP violation in charm is obtained by comparing partial decay widths of charm mesons to those of anticharm mesons. While CPT symmetry requires the total widths of D and \bar{D} to be the same, the partial decay widths $\Gamma(D \rightarrow f)$ and $\Gamma(\bar{D} \rightarrow \bar{f})$ are different in the presence of CP violation, which is signaled by a nonzero value of the asymmetry

$$a_{CP}^f = \frac{\Gamma(D \rightarrow f) - \Gamma(\bar{D} \rightarrow \bar{f})}{\Gamma(D \rightarrow f) + \Gamma(\bar{D} \rightarrow \bar{f})}. \quad (1)$$

This signal is reasonably robust for D^+/D^- mesons, provided that the number of decaying particles and antiparticles is the same. However, at the Large Hadron Collider (LHC), the number of produced D^+ and D^- mesons might not be the same due to the fact that the initial state contains two protons. With CP-violating asymmetries expected to be at the per mille levels [1], it is important to examine the production asymmetry of D mesons both experimentally and theoretically.

Indeed, fixed-target experiments have already observed large asymmetries of charmed mesons and baryons in the forward region. In hadroproduction, the charmed hadrons are preferentially produced with a light valence quark of the same type as what appears in the hadronic beam, for example [2]. This has been termed the “leading particle effect”. More recently, a similar asymmetry in D^\pm production, defined as

$$A_p = \frac{\sigma(D^+) - \sigma(D^-)}{\sigma(D^+) + \sigma(D^-)}, \quad (2)$$

has been measured in the forward region to be $\sim -1\%$ by the LHCb Collaboration [3]. In Section 2, we report our findings of explaining this D^\pm production asymmetry at LHCb by the heavy quark recombination mechanism [4]. In Section 3, using the heavy quark recombination mechanism, we predict the production asymmetries of Λ_c^+/Λ_c^- and $\Lambda_b^0/\bar{\Lambda}_b^0$ for pp collisions at 7 TeV and 14 TeV in the forward region [5]. We conclude in Section 4.

2 D^\pm production asymmetry from heavy quark recombination mechanism

Factorization theorems of perturbative QCD [6] state that heavy hadron production cross section can be written in a factorized form. At the LHC, the cross section for producing a D ($c\bar{q}$) meson in a pp collision, at leading order in a $1/p_T$ expansion, is given by

$$d\sigma[pp \rightarrow D + X] = \sum_{i,j} f_{i/p} \otimes f_{j/p} \otimes d\hat{\sigma}[ij \rightarrow c + X] \otimes D_{c \rightarrow D}, \quad (3)$$

where $f_{i/p}$ is the parton distribution function for parton i in the proton, $d\hat{\sigma}(ij \rightarrow c+X)$ is the partonic cross section and $D_{c \rightarrow D}$ is the fragmentation function describing hadronization of a c quark into a D meson. The corresponding equation for \overline{D} is obtained by replacing c by \bar{c} and D by \overline{D} . Owing to charge conjugation symmetry and that $f_{c/p} = f_{\bar{c}/p}$, perturbative QCD Eq. (3) predicts that $A_p = 0$, which is at least true at leading order in the $1/p_T$ expansion.

To reconcile the experimental observations with QCD, we note that there are corrections to Eq. (3) that scale as powers of Λ_{QCD}/m_c and Λ_{QCD}/p_T . In principle, one can expect nonvanishing power-suppressed contributions to A_p at low p_T . A QCD-based model for these power corrections is the heavy quark recombination mechanism [7, 8, 9, 10]. In this scenario, a light quark involved in the hard scattering process combines with the heavy quark produced in that interaction to form the final state meson, leading to corrections of order $\Lambda_{\text{QCD}}m_c/p_T^2$. This contribution to the cross section is given by (Fig. 1 (a))

$$d\hat{\sigma}[\overline{D}] = d\hat{\sigma}[qg \rightarrow (\bar{c}q)^n + c]\rho[(\bar{c}q)^n \rightarrow \overline{D}], \quad (4)$$

where $(\bar{c}q)^n$ indicates that the light quark of flavor q with momentum of order Λ_{QCD} in the \bar{c} rest frame is produced in the state n , where n labels the color and angular momentum quantum numbers of the quark pair. The cross section is factored into a perturbatively calculable piece $d\hat{\sigma}[qg \rightarrow (\bar{c}q)^n + c]$ and a nonperturbative factor $\rho[(\bar{c}q)^n \rightarrow \overline{D}]$ encoding the probability for the quark pair with quantum number n to hadronize into a final state including a \overline{D} . The perturbative piece was calculated to lowest order in [7]. Equation (4) must then be convoluted with the proton parton distribution functions to get the final hadronic cross section. Besides the $qg \rightarrow (\bar{c}q)^n + c$ process, there are also contributions from $q\bar{c} \rightarrow (\bar{c}q)^n + g$, as shown in Fig. 1 (b). Using the method introduced in [7], we calculate the partonic cross sections from initial state charm [4].

The c quark in Eq. (4) could fragment into a D meson. Thus, to get the full rate due to recombination for producing \overline{D} mesons, we also need to account for the contribution where a light antiquark comes from the proton, while the \bar{c} fragments into a \overline{D} . We thus have three contributions:

$$(a) \quad d\hat{\sigma}[\overline{D}] = d\hat{\sigma}[qg \rightarrow (\bar{c}q)^n + c]\rho[(\bar{c}q)^n \rightarrow \overline{D}], \quad (5a)$$

$$(b) \quad d\hat{\sigma}[\overline{D}] = d\hat{\sigma}[q\bar{c} \rightarrow (\bar{c}q)^n + g]\rho[(\bar{c}q)^n \rightarrow \overline{D}], \quad (5b)$$

$$(c) \quad d\hat{\sigma}[\overline{D}] = d\hat{\sigma}[\bar{q}g \rightarrow (c\bar{q})^n + \bar{c}]\rho[(c\bar{q})^n \rightarrow H] \otimes D_{\bar{c} \rightarrow \overline{D}}, \quad (5c)$$

where H can be any hadron. The recombination cross section for producing a D is obtained by taking the charge conjugate of the above equations. Below, we will neglect C violation and take $\rho[(\bar{c}q)^n \rightarrow \overline{D}] = \rho[(c\bar{q})^n \rightarrow D]$. For simplicity, in process (c) we will restrict H to be D or D^* only and sum over $\bar{q} = \bar{u}, \bar{d}$ and \bar{s} with $SU(3)$ flavor symmetry assumed.

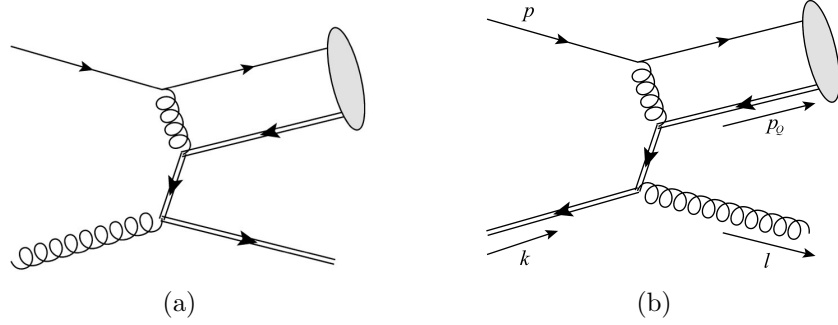


Figure 1: Diagrams for production of a \bar{D} meson by the heavy quark recombination mechanism for (a) $qg \rightarrow (\bar{c}q)^n + c$ and (b) $q\bar{c} \rightarrow (\bar{c}q)^n + g$. Each process has five diagrams. Single lines represent light quarks, double lines heavy quarks, and the shaded blob the \bar{D} meson.

The leading contributions to productions of D^\pm mesons by heavy quark recombination consists of four possible options of n :

$$\begin{aligned} \rho_1^{sm} &= \rho[\bar{c}d(^1S_0^{(1)}) \rightarrow D^+], & \rho_1^{sf} &= \rho[\bar{c}d(^3S_1^{(1)}) \rightarrow D^+], \\ \rho_8^{sm} &= \rho[\bar{c}d(^1S_0^{(8)}) \rightarrow D^+], & \rho_8^{sf} &= \rho[\bar{c}d(^3S_1^{(8)}) \rightarrow D^+]. \end{aligned} \quad (6)$$

These nonperturbative parameters must be extracted from data. Neglecting ρ_1^{sf} and ρ_8^{sf} , the combination $\rho_1^{sm} + \rho_8^{sm}/8$ was determined to be 0.15 by fitting to the E687 and E691 fixed-target photoproduction data [8]. Neglecting ρ_8^{sm} , ρ_1^{sf} and ρ_8^{sf} , the parameter ρ_1^{sm} was determined to be 0.06 by fitting to data from the E791 experiment [9]. In this paper, we take $\rho_1^{sm} \sim 0.06$ and $\rho_8^{sm} \sim 0.7$. It turns out that these two contributions only account for $\sim 30\%$ of the measured asymmetry $A_p = (-0.96 \pm 0.26 \pm 0.18)\%$ at LHCb in Ref. [3]. Therefore, we include ρ_1^{sf} and ρ_8^{sf} and choose values of similar size as the spin-matched parameters. We also include feed down from $D^{*\pm}$. From heavy quark spin symmetry, we have

$$\begin{aligned} \rho[\bar{c}d(^1S_0^{(c)}) \rightarrow D^+] &= \rho[\bar{c}d(^3S_1^{(c)}) \rightarrow D^{*+}], \\ \rho[\bar{c}d(^3S_1^{(c)}) \rightarrow D^+] &= \rho[\bar{c}d(^1S_0^{(c)}) \rightarrow D^{*+}]. \end{aligned} \quad (7)$$

We use MSTW 2008 LO central PDFs with $m_c = 1.275$ GeV and the Peterson parametrization for the fragmentation function is used for $D_{c \rightarrow H}$:

$$D_{c \rightarrow H}(z) = \frac{N_H}{z \left(1 - \frac{1}{z} - \frac{\epsilon_c}{1-z}\right)^2}. \quad (8)$$

$\epsilon_c \sim (m_q/m_c)^2$ was measured to be 0.062 ± 0.007 for the D^{*+} meson [11]. Charge conjugation symmetry and approximate heavy quark symmetry implies that ϵ_c is

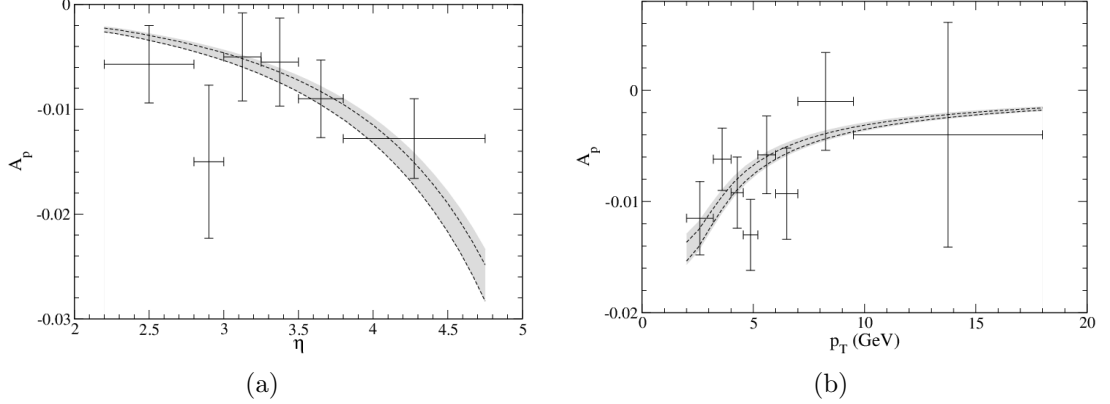


Figure 2: Asymmetry in D^\pm production A_p as a function of (a) pseudorapidity η and (b) transverse momentum p_T in 7 TeV pp collisions. The data points are from LHCb [3]. The grey band is obtained by varying the ρ 's in the intervals $0.055 < \rho_1^{sm} < 0.065$, $0.65 < \rho_8^{sm} < 0.8$, $0.24 < \rho_1^{sf} < 0.30$ and $0.24 < \rho_8^{sf} < 0.30$ respectively. The dashed lines are from varying $0.055 < \epsilon_c < 0.069$.

approximately the same for D^\pm and $D^{*\pm}$. We will take $\epsilon_c = 0.06$ for both D^\pm and $D^{*\pm}$. N_H are determined by the averages of the measured fragmentation probabilities listed in [12]. For the perturbative QCD rate, Eq. (3), which has no asymmetry if we ignore C violation but enters into the denominator of Eq. (2), we use the LO cross section. The factorization scale is set to be $\mu_f = \sqrt{p_T^2 + m_c^2}$.

When integrated over $2 \text{ GeV} < p_T < 18 \text{ GeV}$ and $2.2 < \eta < 4.75$, excluding the region with $2 \text{ GeV} < p_T < 3.2 \text{ GeV}$, $2.2 < \eta < 2.8$, the asymmetry A_p for D^\pm is found to be $-0.88\% < A_p < -1.07\%$ with $0.055 < \rho_1^{sm} < 0.065$, $0.65 < \rho_8^{sm} < 0.8$, $0.24 < \rho_1^{sf} < 0.30$ and $0.24 < \rho_8^{sf} < 0.30$. Figure 2 shows A_p as a function of pseudorapidity η and transverse momentum p_T of the D^\pm mesons as predicted by the heavy quark recombination mechanism. Data from Ref. [3] are shown as well. The grey band is from varying the ρ parameters within the ranges above. The dashed line is obtained using the central value of the ρ parameters and varying ϵ_c within its error bars. The calculated distributions are reasonably consistent with the data. Figure 3 shows the independence of A_p on the factorization scale μ_f when μ_f is varied the the range $\frac{1}{2}\sqrt{p_T^2 + m_c^2} < \mu_f < 2\sqrt{p_T^2 + m_c^2}$. The scale dependence is significant ($\sim 100\%$) at the high η and low p_T ends. An NLO calculation for the heavy quark recombination mechanism in the future will presumably reduce this theoretical uncertainty.

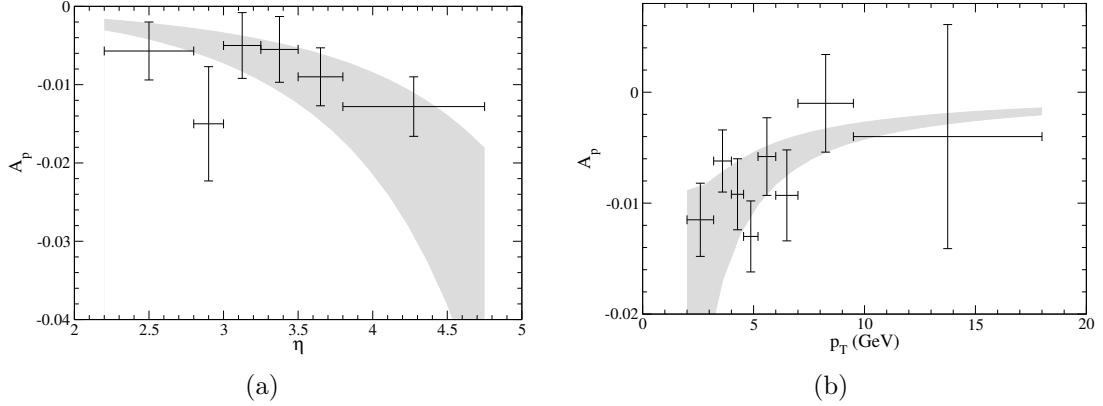


Figure 3: Asymmetry in D^\pm production A_p as a function of (a) pseudorapidity η and (b) transverse momentum p_T in 7 TeV pp collisions. The data points are from LHCb [3]. The grey band is obtained by varying μ_f in the interval $\frac{1}{2}\sqrt{p_T^2 + m_c^2} < \mu_f < 2\sqrt{p_T^2 + m_c^2}$. The ρ 's are taken to be central values of the ranges in Fig. 2. ϵ_c is taken to be 0.06.

3 Λ_c^+/Λ_c^- and $\Lambda_b^0/\bar{\Lambda}_b^0$ production asymmetry from heavy quark recombination mechanism

Similarly, one can apply the heavy quark recombination mechanism to predict production asymmetries for baryons. The contributions to production of Λ_Q (udQ) from recombination are given by

$$(a) \quad d\hat{\sigma}[\Lambda_Q] = d\hat{\sigma}[qq \rightarrow (Qq)^n + \bar{Q}]\eta[(Qq)^n \rightarrow \Lambda_Q], \quad (9a)$$

$$(b) \quad d\hat{\sigma}[\Lambda_Q] = d\hat{\sigma}[Qq \rightarrow (Qq)^n + g]\eta[(Qq)^n \rightarrow \Lambda_Q], \quad (9b)$$

$$(c) \quad d\hat{\sigma}[\Lambda_Q] = \sum_n d\hat{\sigma}[qq \rightarrow (\bar{Q}q)^n + Q] \sum_{\bar{H}_{meson}} \rho[(\bar{Q}q)^n \rightarrow \bar{H}_{meson}] \otimes D_{Q \rightarrow \Lambda_Q}, \quad (9c)$$

$$(d) \quad d\hat{\sigma}[\Lambda_Q] = \sum_n d\hat{\sigma}[\bar{q}g \rightarrow (\bar{Q}\bar{q})^n + Q] \sum_{\bar{H}_{baryon}} \eta[(\bar{Q}\bar{q})^n \rightarrow \bar{H}_{baryon}] \otimes D_{Q \rightarrow \Lambda_Q}. \quad (9d)$$

Here we include both mesons and baryons in the opposite-side recombination (c) and (d). For simplicity, we will take H to be a low-lying heavy hadron. Thus, for Λ_c production we will take H_{meson} to be either D or D^* , and H_{baryon} be any baryon from the lowest mass $J^p = \frac{1}{2}^+$ and $\frac{3}{2}^+$ heavy baryon $SU(3)$ flavor multiplets, and similarly

for Λ_b production. We will also assume $SU(3)$ flavor symmetry. All the ρ 's and η 's scale as Λ_{QCD}/m_Q . The leading nonperturbative parameters $\eta[(Qq)^n \rightarrow \Lambda_Q]$ are

$$\begin{aligned}\eta_3 &= \eta[Qq(^1S_0^{(\bar{3})}) \rightarrow \Lambda_Q], & \tilde{\eta}_3 &= \eta[Qq(^3S_1^{(\bar{3})}) \rightarrow \Lambda_Q], \\ \eta_6 &= \eta[Qq(^1S_0^{(6)}) \rightarrow \Lambda_Q], & \tilde{\eta}_6 &= \eta[Qq(^3S_1^{(6)}) \rightarrow \Lambda_Q].\end{aligned}\quad (10)$$

Contributions of feeddown from heavier baryons in processes (a) and (b) can be taken into account by using the inclusive parameter η_{inc} :

$$\eta_{inc}[(Qq)^n \rightarrow \Lambda_Q] = \eta[(Qq)^n \rightarrow \Lambda_Q] + \sum_{H_{baryon} \neq \Lambda_Q} \eta[(Qq)^n \rightarrow H_{baryon}] B[H_{baryon} \rightarrow \Lambda_Q + X] \quad (11)$$

Here again we assume H_{baryon} is a member of the lowest mass $J^P = \frac{1}{2}^+$ and $\frac{3}{2}^+$ heavy baryon $SU(3)$ flavor multiplets.

For the four ρ 's for D , we use the ranges in Section 2. Best single-parameter fit to Λ_c^\pm asymmetry in fixed target experiments gives $\tilde{\eta}_{3,inc} = 0.058$ for Λ_c [10]. We will take $\eta_{3,inc} = \eta_{6,inc} = \tilde{\eta}_{6,inc} = 0$ and $0.052 < \tilde{\eta}_{3,inc} < 0.064$ for Λ_c . For the η s for Λ_b and ρ s for B , we simply multiply the Λ_c and D counterparts by the theoretical scaling factor m_c/m_b . We use MSTW 2008 LO central PDFs with $m_c = 1.275$ GeV and $m_b = 4.18$ GeV. The fragmentation function $D_{Q \rightarrow \Lambda_Q}$ is taken as

$$D_{Q \rightarrow \Lambda_Q}(z) = f_{\Lambda_Q} \delta(1 - z), \quad (12)$$

where f_{Λ_Q} is the inclusive fragmentation probability. This form of fragmentation function was found to be better than the Peterson form when fitting to fixed target Λ_c^+/Λ_c^- asymmetry data [10]. We take $f_{\Lambda_c^+} = 0.101$, which is the average of the values listed in [12]. $f_{\Lambda_b^0}$ is taken to be 0.09 from [13]. The factorization scale is set to be $\mu_f = \sqrt{p_T^2 + m_Q^2}$. Figures 4 and 5 show the rapidity and transverse momentum distributions of A_p for Λ_c^+/Λ_c^- and $\Lambda_b^0/\bar{\Lambda}_b^0$ in pp collisions at 7 TeV and 14 TeV respectively in the forward region. The asymmetry is significant at the high-rapidity and low- p_T ends ($\sim 2 - 15\%$).

Shown in Fig. 6 are the CMS data [14], which are the rapidity and transverse momentum distributions of $\sigma(\bar{\Lambda}_b^0)/\sigma(\Lambda_b^0)$ for 7 TeV pp collisions in the kinematic region $0 < y < 2$ and $10 \text{ GeV} < p_T < 50 \text{ GeV}$. The CMS data, despite of the large error bars, do have a slight trend of surplus of Λ_b^0 over $\bar{\Lambda}_b^0$ in the regions of high rapidity and low transverse momentum respectively. However, with the values of η 's we used above, the asymmetry predicted from the heavy quark recombination mechanism is negligible in this kinematic region. To see the limit of the heavy quark recombination mechanism in explaining the data, in Fig. 6 we also plot the prediction with larger values of η 's. Here all η_{inc} 's are set equal to each other, with the range being $\Lambda_{QCD}/m_b \sim 0.2 < \eta_{inc} < 1$. The ranges of the ρ s are as those used in Fig. 5.

Although the prediction shows a significant asymmetry $\sim 10\%$ at the high-rapidity and low- p_T ends, it still fails to hit the bin with the largest rapidity. We believe that our previous predictions with smaller η_{inc} 's (Figs. 4-5) are more reasonable since those values of η 's were obtained from better fit to fixed-target experiments. Moreover, the surplus of $\bar{\Lambda}_b^0$ over Λ_b^0 at $y \sim 1$ and $p_T > 20$ GeV in the CMS data cannot be explained by any existing model. We hope that data from LHCb in the future will settle the issue.

4 Conclusion

In summary, we have calculated the D^\pm asymmetry using the heavy quark recombination mechanism for production at the LHCb experiment. The measured asymmetry of $A_p = -0.96 \pm 0.26 \pm 0.18\%$ in the kinematic range $2.0 \text{ GeV} < p_T < 18 \text{ GeV}$ and $2.2 < \eta < 4.75$ [3] can be reproduced using reasonably sized nonperturbative parameters $\rho_{1,8}^{sm,sf}$. Further, the p_T and η distributions are simultaneously reproduced by the heavy quark recombination mechanism.

We have also used the heavy quark recombination mechanism to calculate the production asymmetries for Λ_c^+/Λ_c^- and $\Lambda_b^0/\bar{\Lambda}_b^0$ at the LHCb experiment. The differential distributions are significant at the high-rapidity and low- p_T ends ($\sim 2 - 15\%$). The integrated asymmetries in the LHCb region are of the order of $\sim 1 - 3\%$ and should be measurable.

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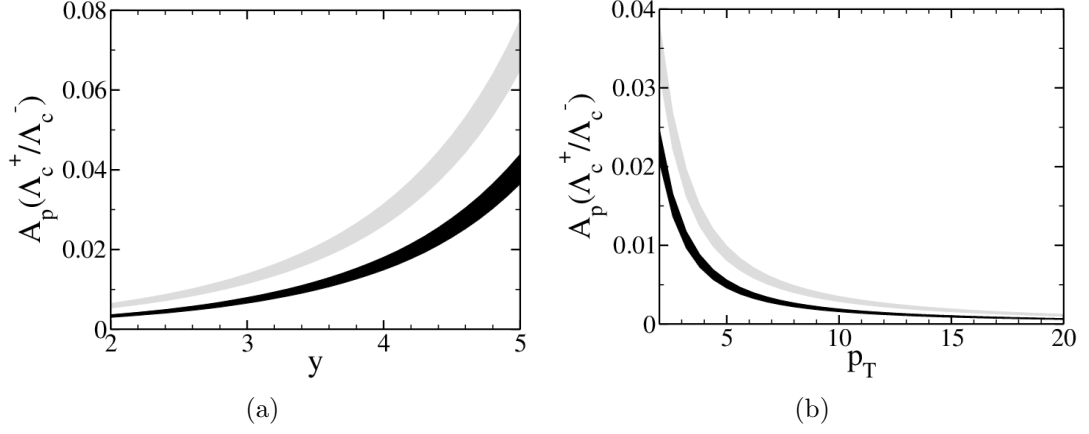


Figure 4: Asymmetry in Λ_c^+/Λ_c^- production as a function of (a) rapidity y and (b) transverse momentum p_T in the kinematic region $2 < y < 5$ and $2 \text{ GeV} < p_T < 20 \text{ GeV}$ in 7 TeV (grey band) and 14 TeV (black band) pp collisions. The integrated A_p is found to be $2.0\% < A_p(\Lambda_c^+/\Lambda_c^-) < 2.4\%$ for $\sqrt{s} = 7 \text{ TeV}$ and $1.2\% < A_p(\Lambda_c^+/\Lambda_c^-) < 1.5\%$ for $\sqrt{s} = 14 \text{ TeV}$.

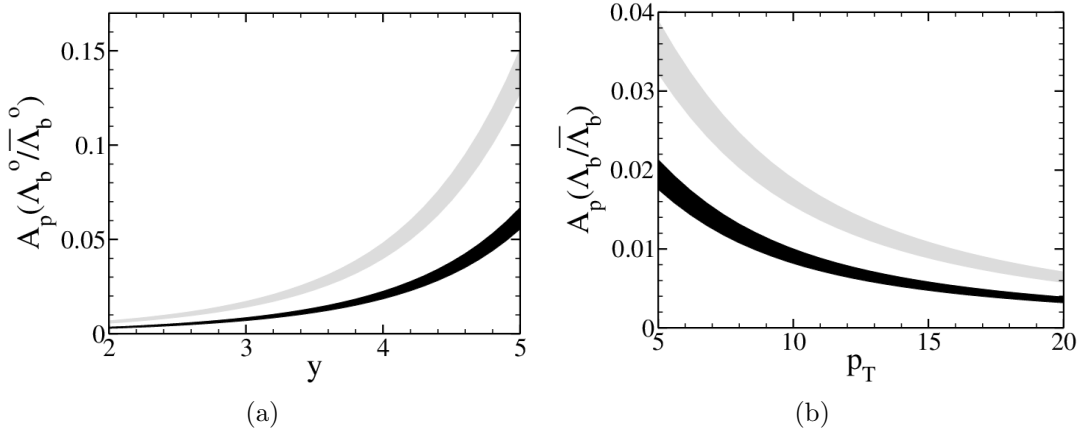


Figure 5: Asymmetry in $\Lambda_b^0/\bar{\Lambda}_b^0$ production as a function of (a) rapidity y and (b) transverse momentum p_T in the kinematic region $2 < y < 5$ and $5 \text{ GeV} < p_T < 20 \text{ GeV}$ in 7 TeV (grey band) and 14 TeV (black band) pp collisions. The integrated A_p is found to be $2.2\% < A_p(\Lambda_b^0/\bar{\Lambda}_b^0) < 2.6\%$ for $\sqrt{s} = 7 \text{ TeV}$ and $1.1\% < A_p(\Lambda_b^0/\bar{\Lambda}_b^0) < 1.4\%$ for $\sqrt{s} = 14 \text{ TeV}$.

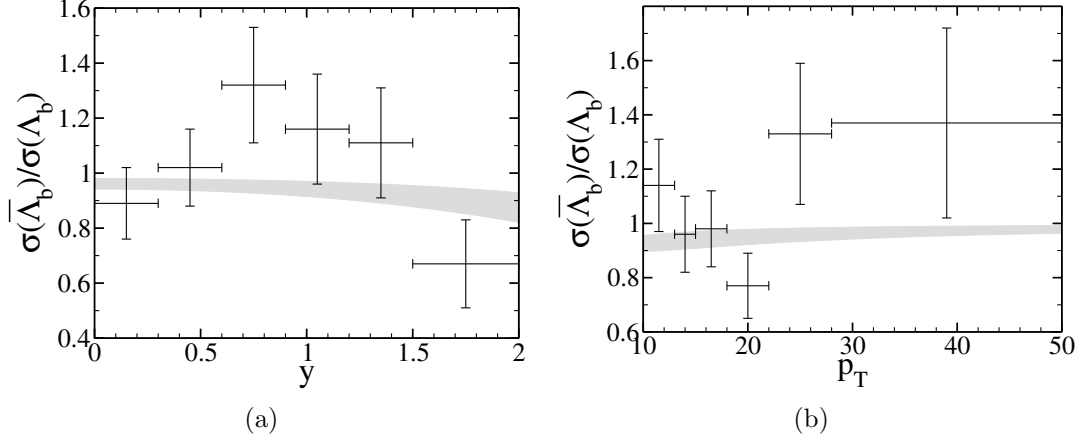


Figure 6: $\sigma(\bar{\Lambda}_b^0)/\sigma(\Lambda_b^0)$ as a function of (a) rapidity y and (b) transverse momentum p_T in the kinematic region $0 < y < 2$ and $10 \text{ GeV} < p_T < 50 \text{ GeV}$ for 7 TeV pp collisions. The data are from CMS [14]. The grey band is our prediction from the heavy quark recombination mechanism with all η_{inc} 's set equal to each other, the range being $0.2 < \eta_{inc} < 1$. The ranges of ρ 's are as those used in Fig. 5.

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